# Occupational Choice and Redistributive Taxation

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**Abstract**

This paper studies the role of redistributive taxation in a simple model of occupational choice where credit markets are imperfect. In such a set-up family income determines whether children will invest or not in schooling and so whether they will be qualified or not. The *laisser-faire* equilibrium is characterized by permanent inequality. Numerical work shows that, in comparison to the pure equilibrium, distortive taxation that is used to finance the educational subsidy and redistribution increases the ratio of skilled labor. The whole dynamics of the transitional period are studied numerically.

**Keywords**: Inequality, borrowing constraints, redistribution, educational choice.
**JEL Classification Numbers**: H31; D31; D58

## 1 Introduction

Human capital is one of the main determinants of growth according to the new theories of endogenous growth (e.g., Lucas (1988), Romer (1990), Mankiw *et al.* (1992)). The essential component of human capital is schooling. Empirical works suggest that formal schooling is an important determinant of productivity levels. For instance, Mankiw *et al.* (1992), Benhabib

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and Spiegel (1994)[1], Barro (1999) and Aghion *et al.* (2004) find that school enrollment is positively correlated with GDP per worker.

The partially private character of schooling makes its financing important. If financial markets are complete and perfect, then anyone can borrow–if necessary–and invest in schooling. But if not, then according to the extent of imperfection, fewer agents are able to realize this investment. In the widely assumed case, impossibility of borrowing against future labor income (which is surely the case in most of the developing countries), parental wealth will be determining for the choice to continue in education or not. In such an environment the government may improve the resource allocation by fiscal instruments. Usually, there are two forms of the government intervention in the schooling process. In one hand, the schooling is furnished publicly, people do not pay at all or pay very little, and in the other one, there are student loans, tax credits for schooling expenditures[2]. In the first case, the nature of the intervention impose its use: the ones who want to benefit from these policies have no choice. But in the second case, these are almost always parents who receive schooling subsidies/funds and they have the opportunity to use these funds for other uses than schooling. As a result their result may differ in terms of efficiency and the question "how to spend the marginal government income?" becomes crucial.

To study the importance of parental income and alternative public policies, I develop a deterministic two-period overlapping generations model of heterogenous agents with imperfect credit markets and redistribution. The imperfection is such that the young generation can not borrow against its future income. The heterogeneity consists of the initial distribution of bequests. Given credit market imperfection, there can be some agents who are credit constrained in their first period or their life, because investment in schooling is realized when young. In order to prevent this imperfection to generate suboptimal equilibria the government would like to intervene. To capture the realistic part of the story, the redistribution is realized by two

[1]It is ironic to see that the study of Benhabib and Spiegel (1994) is cited as both for and counter the fact that human capital affects the growth rate. In fact, when human capital is considered as an input like row labor and physical capital (Becker view), like in Mankiw *et al.* (1992), they find that the effect of human capital on per capita growth rate is insignificant and almost negative. See also Romer (1989) and Krueger and Lindahl (2001) for a similar result. In the alternative formulation where human capital influences productivity/technological progress (Nelson-Phelps view), like in Romer (90) and Aghion *et al.* (2004), their conclusion is that human capital affects positively per capita growth rate.

[2]Hendel *et al.* (2005, p.861) report that the ratio of student loans to the Federal GDP was 1 % in 1965 while it attains 25 % in 1995.

fiscal instruments in this paper: educational and fiscal redistribution.

It is shown that parental income/wealth distribution is the main determinant in the decision whether or not to invest in schooling. Then, intuitively, a fiscal policy that lessens the borrowing constraints may increase the number of agents who are able to invest in schooling. Since agents are heterogenous in initial wealth and credit markets are imperfect, the dynamics of macroeconomic aggregates depend on the whole history. I am not able to get analytically tractable expressions for the key variables like capital stock, skilled agents' ratio etc... This is why the paper uses numerical methods to get insights about the evolution of the variables of interest.

Agents are identical except the parental bequest they receive. The initial distribution of bequests is assumed to be log-normal. Previous works of Chiu (1998), Owen and Weil (1998), and Maoz and Moav (1999) about inequality and borrowing constraints have used a similar[3] set-up. They show that there is a threshold of bequest–call it $b^*$–such that the agents who get a bequest lower than $b^*$ will not invest in schooling for a given wage premium. Thus, we are in front of a polar case; given the level of parental bequest either we will be skilled or unskilled; all skilled agents are relatively rich and thus *unconstrained* in the credit market while the unskilled ones are poor and *constrained*. The problem is that this parallelism between educational and financial situations is not satisfactory. It would be more appropriate to think that there are agents who get a transfer a little bit higher than $b^*$ but who do not prefer to invest in schooling, i.e. unskilled and unconstrained.

In order to solve this problem I introduce the saving mechanism which is assumed absent in an *ad hoc* manner in Chiu (1998), and Maoz and Moav (1999) but also assume that agents work in the second period of their life (which is not the case in Owen and Weil (1998)). Hence, the model is such that agents receive a bequest but do not work in the first part of their life; they consume and decide whether to invest in schooling or not. In the second period, they consume, make a transfer to their descendant. I obtain a richer set-up; we have, *ex ante*, four type of agents (constrained-skilled, constrained-unskilled, unconstrained-skilled, and unconstrained-unskilled ones) at any moment. The dynamics of the economy are more complex and realistic. Depending on the difference between the fixed cost of education and wage premium, there are two regimes. If this difference is low enough there will be exactly four types of agents, more importantly now we will have un-

[3]Differently from this study, all these three papers assume that the talent of agents is stochastic. In Chiu, and Maoz and Moav, more importantly, there is no intertemporal trade (saving) between two periods. While in Owen and Weil there is saving but agents work only in the first period.

skilled and unconstrained agents with a bequest level slightly higher than the threshold. Otherwise, there will be only three types: unconstrained-skilled, constrained-skilled and constrained-unskilled ones. In this last case as well, we have a new type–constrained-skilled–that does not exist in the above cited papers.[4].

Numerical analysis shows that a fiscal policy consisting of a schooling subsidy and redistribution may increase the ratio of skilled agents in the economy. In comparison to the pure equilibrium, distortive taxation that is used to finance educational redistribution or fiscal redistribution increases the ratio of skilled labor. Yet, the education subsidies are more efficient[5]. The intuition for such a result is that direct redistribution diminishes also incentives for schooling investment.

Another related paper is Galor and Zeira (1993) even if the imperfection nature is different from the cited papers. Whereas, there are numerous common results: they show that when investment in human capital is indivisible and the credit markets are imperfect, initial conditions affect not only the short-run but also the long-run variables. Particularly, they show that multiple equilibria are possible and income distribution is not ergodic so that agents will be divided into subgroups such as rich and poor ones. This is the result that I obtain in pure equilibrium case; in the long-run we have two group of constrained agents; the unskilled (relatively poor) and skilled (relatively rich) ones. Further, I extend their work by studying the transitional dynamics of a similar model. For example, what is the ratio/number of the skilled agents in, say period $i$–where $i$ can take any value–is not studied in their work. Thanks to numerical work in the section 5, we are able to respond such a question.

Chiu (1998) uses a 2 period overlapping generations model like Galor and Zeira (1993) to study how parental income may affect occupational choice of children. The novelty is that ability is stochastic. The main finding is that a mean preserving improvement in distribution of income increases the number of qualified people. Numerical simulations in the section 5 confirm Chiu's theoretical findings; redistribution (either fiscal or educational) increases the number of skilled agents.

Owen and Weil (1998) and Maoz and Moav (1999) study on interaction between mobility and inequality and growth. Both works find that remov-

[4]From the point of view of this typology, the present model is more closer to the one of Galor and Zeira (1993) even if the nature of imperfection is totally different in their paper (the borrowing and lending rates are different). In that paper, borrowers are surely skilled while lenders may be either skilled or unskilled.

[5]See also Bénabou (2002) for the same argument in a different set-up.

ing barriers to the schooling by lessening borrowing constraints increases output/consumption per capita by increasing the ratio of skilled labor. In both papers ability is stochastic and there are liquidity constraints. Another central difference between these works and this paper lies in the production function specification. They assume that skilled and unskilled are complements while in this paper they are perfect substitutes. Hence, as the number of skilled agents increases, the wage gap diminishes in their work and too much redistribution removes all incentives to be qualified. But in the present work, this effect does not exist. This is actually the cost I pay in order to have a model that can be simulated. However, from both an empirical (see for example Autor et al. (1998)) and theoretical (see for example Lucas (1988) or Acemoğlu (1998)) point of view an increase in the number of skilled workers does not necessarily decrease the wage premium.

Owen and Weil (98) focus on the mobility and stability in the steady-states while my analysis shows the complete trajectory of the variables of interest. Maoz and Moav (1999) assumes that all skilled and unskilled agents are homogenous among themselves. The reason that pushes them to a such hypothesis is that there is no capital markets, i.e. no saving[6]. They do not analyze explicitly how the distribution of income will evolve in time, while I do in this paper.

The main assumption in all these cited [theoritical] works and this paper is that credit markets are imperfect. The empirical works of Haveman and Wolfe (1995), Acemoğlu and Pischke (2001), Carneiro and Heckman (2002) and Plug and Vijverberg (2005) (based on US data); Blanden et al. (2003) and Blanden and Gregg (2004) (based on British data), suggest that there are large effects of family income on enrollments and schooling attainment.

The present paper is organized as follows. Section 2 describes briefly the model. Section 3 studies the occupational choice under borrowing constraints and related regimes under which the economy operates. Section 4 characterizes the equilibrium and wealth dynamics of the economy. Section 5 gives some numerical results about the role of taxation and redistribution and finally section 6 concludes.

[6] They affirm (footnote 16, p.683) that [even if]

> ...workers belonging to the same group can differ, both in the transfer they received from their parents, and in their abilities. However, these historic differences are isolated from current decisions on account of no-lending assumption.

## 2 Model

I build a simple model of investment in schooling and intergenerational persistence of income inequality. The model economy consists of two-period overlapping generations. The parents are either skilled or unskilled. Labor supply is inelastic and wages are determined in a competitive labor market. Each agent has one unit of time endowment. The representative firm has access to a constant returns to scale production technology with capital, skilled and unskilled labor as the only inputs. Each parent has one child. Every child is characterized by the same ability in order to focus on the role played by family income and borrowing constraints. Parents derive utility from consumption and investment in their children, thus they allocate their total income between consumption and investment in human capital of their children.

Let the production function be,

$$Y_t = \Gamma K_t^{\beta} N_t^{1-\beta} \tag{1}$$

further, consider that $H$ and $L$ are perfect substitutes.

$$N_t = H_t + \theta L_t \tag{2}$$

The reason of this assumption is tractability: I can analyze numerically the path of variables of interest, such as the number of skilled/unskilled agents. However, from both an empirical (see for example Autor et al. (1998)) and theoretical (see for example Lucas (1988) or Acemoğlu (1998, 2002)) point of view this assumption makes sense because an increase in the number of skilled workers increases the wage premium. But, contrary to the usual assumption in related works on occupational choice under borrowing constraints[7], an increase in the number of skilled workers certainly does not decrease it.

Schooling takes one period, today's skilled workers have gone to the school the previous period. In a constant population the sum of unskilled workers and skilled workers will be constant.

$$H_t + L_t = 1 \tag{LME}$$

### 2.1 Producers

For tractability, assume that the production function is given by (1). In a competitive environment, the factor demand is given by profit maximization.

[7] See for example Owen and Weil (1998) and Maoz and Moav (1999).

The marginal cost will be equal to the marginal benefit. Assume that our model is one of small open economy. Given perfect mobility of capital, the world interest rate is given and constant, i. e. $r_t = r, \forall t$. Normalizing the price of the consumption good to unity and defining $k_t = K_t/N_t$, $1 + r = R$ we can write the maximization program of the firm like

$$\underset{K,H,L}{\text{Max}}\ \pi = \Gamma K_t^{\beta}(H_t + \theta L_t)^{1-\beta} - RK_t - w_t^H H_t - w_t^L L_t$$

The first order conditions (FOCs) for the firm are

$$\begin{aligned} R &= \beta \Gamma k_t^{\beta-1} \\ w_t^H &= (1-\beta)\Gamma k_t^{\beta} \\ w_t^L &= \theta(1-\beta)\Gamma k_t^{\beta} \end{aligned} \tag{3}$$

The important point is that if $w^L/w^H \neq \theta$ there will be only one type in our economy. To have both types we need $w^L/w^H = \theta$.

## 2.2 Consumers

I assume that each agent has a single parent and a single child. She lives only two periods. In the first one, she does not work, but receives a bequest ($b_t^i$) from her parent. She can use this for consumption ($c_t^i$) or indivisible schooling investment, in order to be skilled. In the second period she works, consumes ($d_{t+1}^i$) and makes a transfer ($b_{t+1}^i$) to her child. I assume that capital markets are imperfect so that we can not borrow when we are young. The government subsidies the education costs at a rate of $\chi_t$. This is what I call the *educational redistribution*. So, the real education costs are equal to $f_t(1-\chi_t)$. Finally, assume that schooling cost is correlated with skilled wage of the following period $f_t = f w_{t+1}^H$. The reasons for such an assumption are twofold. The first one is intuitive: the gain/profit of any action is positively correlated to its total costs. So, it is natural to assume that the implied cost of schooling are proportional to its opportunity, skilled wage. The second one is that this yields tractable formulas that can be interpreted clearly and easily in comparison to the alternative formulations that add no more insights but complicate the presentation.

So we can write the budget constraint of a member of generation $t$ like

$$\begin{aligned} c_t^i + s_t^i &= b_t^i - e f_t(1-\chi_t) \\ d_{t+1}^i + b_{t+1}^i &= W_{t+1}^i + s_t^i R - T_{t+1}^i \end{aligned} \tag{CBCs}$$

where $W_t := ew^H + (1-e)w^L$ is the gross wage income. $e$ is a discrete choice variable. It is equal to 1 if parents decide for schooling and 0 otherwise. In order to have a tractable and simple model, let us assume, a linear but progressive tax scheme

$$T_t^i = \tau_t[W_t + rs_{t-1}^i] - z_t$$

The first important point of this tax function is that I assume, following Sandmo (1983), that the government makes a lump-sum transfer $z_t$ (usually called basic income[8]) for all regardless of his wealth and type. This is what I call the *fiscal redistribution*. Let, this income be proportional to the unskilled workers' wage ratio, i.e. $z_t = \eta_t w_t^L$. The reason of this hypothesis is the following: in real life, the basic income schemes are never higher than the minimum wage[9]; otherwise there would be no worker who work for the minimum wage. This specification will permit us to compare the fiscal redistribution to the wage of the unskilled in this paper. The second important point is that the tax rate on labor income and capital income from both domestic and foreign bonds is at the same, $\tau_t$. This means that the government applies a residence-based income taxation which means that the pre-tax rates of return to capital must be equal between countries.

*Ex ante*, the consumer $i$'s maximization program is the following one

$$\begin{aligned} &\max_{c,d_+,b_+,e} U(c_t^i, d_{t+1}^i, b_{t+1}^i) \\ &c_t^i + s_t^i = b_t^i - ef_t(1-\chi_t) \\ &d_{t+1}^i + b_{t+1}^i = W_{t+1}^i + s_t^i R - T_{t+1}^i \\ &s_t^i \geq 0 \end{aligned} \tag{CP}$$

[8] In fact, this is not the sole way to have a linear and progressive tax system. Another may prefer tax function without lump-sum transfer but with exemption (let $I$ denote income)

$$T(I) = \begin{cases} 0 & \text{if } I < z \\ \tau I & \text{if } I \geq z \end{cases}$$

And finally, tax function can incorporate both a lump-sum transfer and exemption as in d'Autume (2002) where he used this formulation in order to explore the effects of a tax reform on French economy. In this case $z$ is the guaranteed minimum revenue.

$$T(I) = \begin{cases} I - z & \text{if } I < z \\ \tau(I - z) & \text{if } I \geq z \end{cases}$$

[9] One can think RMI (Revenue Minimum d'Insertion) in the French case.

The utility function is logarithmic, this is the simplest well behaved function.

$$U^i(t) = \ln c_t^i + \alpha \ln d_{t+1}^i + (1-\alpha) \ln b_{t+1}^i \tag{4}$$

$b_t$ is the bequest of agent from her parents. I assume that the cumulative distribution function of bequests is given by $G_t$ and the density function by $g_t$. $G_t$ is defined over $\Omega_t$. Further, I assume that the median, is smaller than the mean, $\mu_t$, i.e. $G_t(\mu_t) > 1/2$. At time 0, which I interpret as initial period, I suppose that distribution of bequests is given. But the subsequent distributions will evolve over time.

$$b_t^i \in [\underline{b}_t, \bar{b}_t] \equiv \Omega_t$$

The aggregate (and average) variables of the economy are given by

$$Q_t = \int_0^1 q_t^i di = \int_{\underline{b}}^{\bar{b}} q_t^i dG_t, \quad q = b, s, c, d, b$$

$G_t$ assigns weights to subsets of $\Omega_t$ with $\int_{\underline{b}}^{\bar{b}} dG_t = 1$.

It is well known that the consumer makes a two stage optimization. In the first stage the intertemporal one i.e. for a given first period revenue she chooses her consumption and saving. And in the second stage she makes the intratemporal one, i.e. how to allocate a given revenue between two uses in the second period. Let us call the sum of the two purchase of the second period $x$, so that $x = d + b$. In the second period of her life the agent's program is[10]

$$\underset{d,b}{\text{Max}} \quad \alpha \ln d + (1-\alpha) \ln b \quad \text{s.t.} \quad d + b = x$$

This yields in $d = \alpha x$ and $b = (1-\alpha)x$. Now, I have

$$\alpha \ln d + (1-\alpha) \ln b = \ln x + \mathtt{R1}$$

with $\mathtt{R1} = \alpha \ln \alpha + (1-\alpha) \ln(1-\alpha)$. So, the utility function in terms of $x, c$ is (neglecting the constant $\mathtt{R1}$).

$$U^i = \ln c_t^i + \ln x_{t+1}^i \tag{4'}$$

[10] I will not use the time subscript, if there is no ambiguity.

## 3 Occupational choice

In fact, according to the *separation theorem* one does not need to make an explicit comparison of utility in either cases to determine the agent's schooling decision, in a world where there are no credit constraints. Following this theorem, if credit markets are perfect ($s$ can take any value), then pure investment decisions will be made independently of preferences (or equivalently consumption decisions). The reason is the following: as a discrete choice variable, $e$, does not appear in the utility function, the schooling decision will be made to maximize the budget constraint. To show this point, let us write the Lagrangian as (neglecting `R1`)

$$\begin{aligned}\mathcal{L} = \ln c_t^i + \ln x_{t+1}^i + \lambda_{1t}[b_t^i - e\hat{f}_t - s_t^i - c_t^i] \\ + \lambda_{2t}\Big(\hat{W}_{t+1} + s_t^i \hat{R}_{t+1} + z_{t+1} - x_{t+1}^i\Big) + \zeta_t^i s_t^i\end{aligned} \tag{5}$$

with $\hat{f}_t := (1-\chi_t)f_t$, $\hat{W}_t := (1-\tau_t)W_t$ and $\hat{R}_t := 1 + (1-\tau_t)r$. FOCs yield

$$\begin{aligned}\frac{1}{c^i} &= \lambda_1 \\ \frac{1}{x^i} &= \lambda_2 \\ \lambda_1 &= \lambda_2 \hat{R} + \zeta \\ s^i &\geq 0, \quad \zeta \geq 0, \quad \zeta^i s^i = 0 \\ c^i + s^i &= b^i - e\hat{f} \\ x^i &= \hat{W} + s^i \hat{R} + z\end{aligned} \tag{FOCs}$$

When the positivity constraint of savings is not binding (either the credit markets are perfect so that we may have $s < 0$ or the agent has already a high bequest such that she has positive savings), we have $\zeta = 0$ and the FOCs yield

$$c_t^i = \frac{1}{2}\Big(b_t^i - e\hat{f}_t + \frac{z_{t+1} + \hat{W}_{t+1}}{\hat{R}_{t+1}}\Big) \tag{6a}$$

$$x_{t+1}^i = \hat{R}_{t+1} c_t^i \tag{6b}$$

$$s_t^i = b_t^i - e\hat{f}_t - c_t^i \tag{6c}$$

An agent will choose to become skilled only if

$$U^{iH} \geq U^{iL} \tag{7}$$

The first one is straightforward but the second needs more attention. Using (4′) and (6) we obtain

$$\begin{aligned} U_t^i &= \ln c_t^i + \ln x^i \\ &= 2\ln c^i + \ln \hat{R} \end{aligned} \tag{8}$$

In order to maximize this utility level, the individual $i$ needs to maximize only $c^i$.

$$\underset{e}{\text{Argmax}}\ c^i = \frac{1}{2}\Big[b^i + \frac{z}{\hat{R}} + \frac{(1-\tau)w^L}{\hat{R}} + e\Big(\frac{(1-\tau)(w^H - w^L)}{\hat{R}} - \hat{f}\Big)\Big]$$

We see that for a large wage premium all individuals would like to be skilled. This can be called skill premium condition, and is given by

$$(1-\tau_{t+1})(w_{t+1}^H - w_{t+1}^L) \geq (1-\chi_t)f_t\hat{R}_{t+1} \tag{9}$$

$$\Rightarrow \frac{(1-\tau_{t+1})(1-\theta)}{(1-\chi_t)\hat{R}_{t+1}} > f \tag{SPC}$$

Let us define the threshold $f_t^{**}$ such that SPC is given by equality.

$$f_t^{**} = \frac{(1-\tau_{t+1})(1-\theta)}{(1-\chi_t)\hat{R}_{t+1}}$$

If $f < f^{**}$, then the agents who are able to, would invest in schooling; but, if not, then there will be no skilled agent in the economy. In the $f = f^{**}$ case, the ratio of skilled-unskilled will be indeterminate, because it makes no difference to the agent to be skilled or not.

When the agent is constrained on credit markets, i.e. $s_t^i = 0$, then the parental bequest will determine if she will be qualified or not. To see it mathematically let us rewrite the FOCs of the agent when her constraint of positive saving is binding i.e., $\zeta_t^i > 0$. FOCs, give

$$\zeta^i = \frac{1}{c^i} - \frac{1}{x^i}\hat{R} \tag{10}$$

Putting these results in the budget constraint of the agent one gets

$$\begin{aligned} c^i &= b^i - e\hat{f} \\ x^i &= \hat{W} + z \end{aligned} \tag{11}$$

which means

$$\zeta^i = \frac{1}{b^i - e\hat{f}} - \frac{\hat{R}}{\hat{W} + z} \tag{12}$$

Using (12) to determine $b_t$ which makes $\zeta > 0$ (equivalently $s_t < 0$), we can find

$$b^i < b^x = e\hat{f} + \frac{\hat{W} + z}{\hat{R}}$$

In order to alleviate the burden of notation I will define $\omega_t^H := (1-\tau_t)w^H + z_t$ and $\omega_t^L := (1-\tau_t)w^L + z_t$.

For $e = 0$ we get the threshold below which the agent does not save given that she will be unskilled

$$b_t^x = b_t^p = \frac{\omega_{t+1}^L}{\hat{R}_{t+1}} \tag{13}$$

and for $e = 1$ the threshold below which the agent does not save given that she will be skilled

$$b^x = b_t^r = \hat{f}_t + \frac{\omega_{t+1}^H}{\hat{R}_{t+1}} \tag{14}$$

Using (4′) and (11) one obtains

$$\begin{aligned} U^i &= \ln c^i + \ln x^i \\ &= \ln(b_t^i - e\hat{f}_t) + \ln\left(\hat{W}_{t+1} + z_{t+1}\right) \end{aligned} \tag{15}$$

For instant, we do not know who will invest in schooling and who will not. At most, in our constrained economy *a priori* there are four type of agents, as we see in the Table (1). Given a level of parental bequest, the agent will choose her type. It is, as if there were four utility technologies, $U_1, U_2, U_3$, and $U_4$, that accept parental bequest as sole input. The agent chooses the one that ensures the highest level of utility to her.

| | $e = 0$ | | $e = 1$ | |
|---|---|---|---|---|
| $s > 0$ | type 4: | $U^L > U^H$<br>$b^i > b^p$ | type 3: | $U^H > U^L$<br>$b^i > b^r$ |
| $s = 0$ | type 1: | $U^L > U^H$<br>$b^i \leqq b^p$ | type 2: | $U^H > U^L$<br>$b^i \leqq b^r$ |

Table 1: Possible agent types

Let us write down the utility levels of each type (neglecting `R1` which is common to all):

$$
\begin{aligned}
U_1^i &= \ln b_t^i + \ln \omega_{t+1}^L \\
U_2^i &= \ln(b_t^i - \hat{f}) + \ln \omega_{t+1}^H \\
U_3^i &= 2\ln\left[\frac{1}{2}\Big(b_t^i - \hat{f}_t + \frac{\omega_{t+1}^H}{\hat{R}_{t+1}}\Big)\right] + \ln \hat{R}_{t+1} \\
U_4^i &= 2\ln\left[\frac{1}{2}\Big(b_t^i + \frac{\omega_{t+1}^L}{\hat{R}_{t+1}}\Big)\right] + \ln \hat{R}_{t+1}
\end{aligned}
\tag{16}
$$

The first question is whether there are really four types of agents in our economy. The immediate response is "it depends". It is the non-divisible cost of schooling that determines how many type of agents will be present.

Below, I show that there are two endogenous thresholds, $f^*$ and $f^{**}$ that determines the evolution of the economy. I have already showed that $f^{**}$ is the lowest cost of education such that SPC holds with inequality, i.e. everybody would like to invest in education if she can.

When agents are constrained the schooling choice will be made only if (7), i.e. $U^{iH} \geq U^{iL}$ but also the agent has enough wealth to finance it. Let us call it "feasibility constraint" (FC)

$$
b^i \geq \hat{f} \tag{FC}
$$

This is why I make the following assumption.

**Assumption 1** *Let us assume that, the cost of education is not too high so that even the constraint agents are able to undertake education, i.e. $\hat{f} < b^p$.*

Following the value of $f$ there are two phases in our economy:
**Phase 1**: $f \leq f^*$

**Proposition 1** *When we are in the Phase 1, for given wages and interest rate, there is a bequest level $b^*$ such that the agents $b^i > b^*$ choose to be skilled.*

**Proof.** Consider the agents with $b^i \leq b^p$. The ones with $b^i < \hat{f}$ have no choice than to be unskilled. For the ones $b^i \in (\hat{f}, b^p)$, the agent will choose either $U_1$ or $U_2$. The schooling decision will be made only if $U_2^i - U_1^i > 0$. Comparing the two functions we see that $U_2^i - U_1^i > 0$ implies $b^i > b^*$ with

$$
b_t^* = \frac{(1-\chi_t) f w_{t+1}^H}{1 - \frac{\omega_{t+1}^L}{\omega_{t+1}^H}} \tag{17}
$$

■

It means that, given credit market imperfections, there is a threshold level of bequests, $b_t^*$, under which it is not optimal to invest in education. Only the ones with ($b_t^i \geq b_t^*$) will choose to be skilled, as we see in the Figure (1). The wage premium has a negative effect on schooling investment: $db_t^*/d(w_{t+1}^H/w_{t+1}^L) < 0$ while the schooling cost has a positive one: $db_t^*/df > 0$.

As $\theta < 1$ we have already $b^* \geq \hat{f}$, but one has to verify if $b^* < b^p$. Otherwise we must compare not only $U_2^i - U_1^i$ but also $U_2^i - U_4^i$. A comparison of $b^*$ and $b^p$ shows that, for given wage levels, it is $f$ which determines whether $b^* < b^p$ or not. There is a threshold level, say $f^*$, such that when $f \leqq f^*$ we have $b^* \leqq b^p$. This level is given by

$$f_t^* = \Big(1 - \frac{\omega_{t+1}^L}{\omega_{t+1}^H}\Big)\frac{\omega_{t+1}^L}{(1-\chi_t)w_{t+1}^H \hat{R}_{t+1}}$$

Now, it is simple to define $f^*$: the threshold for constrained agents to be able to invest in schooling.

**Phase 2**:$f^* < f \leq f^{**}$:

**Proposition 2** *When we are in the Phase 2, for given wages and interest rate, there is a bequest level $b^{**}$ such that the agents $b^i > b^{**}$ choose to be skilled.*

**Proof.**

Since $f > f^*$, all agents with $b^i \leq b^p$ will choose to be unskilled (given that $b^* > b^p$). The agents with a bequest $b^i > b^p$ will invest in schooling only if $U_2^i - U_4^i > 0$. For low $b^i$, we have $U_2 - U_4 < 0$ (think of, for example, $b^i$ near $\hat{f}$ which implies that $U_2$ goes to $-\infty$). On the other hand, for $b^i = b^r$ we know that $U_2 = U_3 \geq U_4$ given the condition SPC. We know also that the function $U_2 - U_4$ is continuous in $b^i$ (for $b^i > \hat{f}$) and increasing for relatively low values of $b^i$, $\frac{d\,(U_2-U_4)}{d\,b^i} = \frac{1}{b^i - \hat{f}} - \frac{2}{\omega^L/\hat{R}+b^i} > 0$, but decreasing for high values of $b^i$, $\frac{d\,(U_2-U_4)}{d\,b} < 0$. Thus, $U_2 - U_4$ is concave and there are two roots, $b_1^{**}, b_2^{**}$ of which $b_1^{**}$ is relevant. See the Figure (1) for a graphical representation.

$$b_{1t}^{**} = \frac{2\,\omega_{t+1}^H - \omega_{t+1}^L - 2\sqrt{\omega_{t+1}^H\,[\omega_{t+1}^H - \hat{f}_t\,\hat{R}_{t+1} - \omega_{t+1}^L]}}{\hat{R}_{t+1}}$$

$$b_{2t}^{**} = \frac{2\,\omega_{t+1}^H - \omega_{t+1}^L + 2\sqrt{\omega_{t+1}^H\,[\omega_{t+1}^H - \hat{f}_t\,\hat{R}_{t+1} - \omega_{t+1}^L]}}{\hat{R}_{t+1}}$$

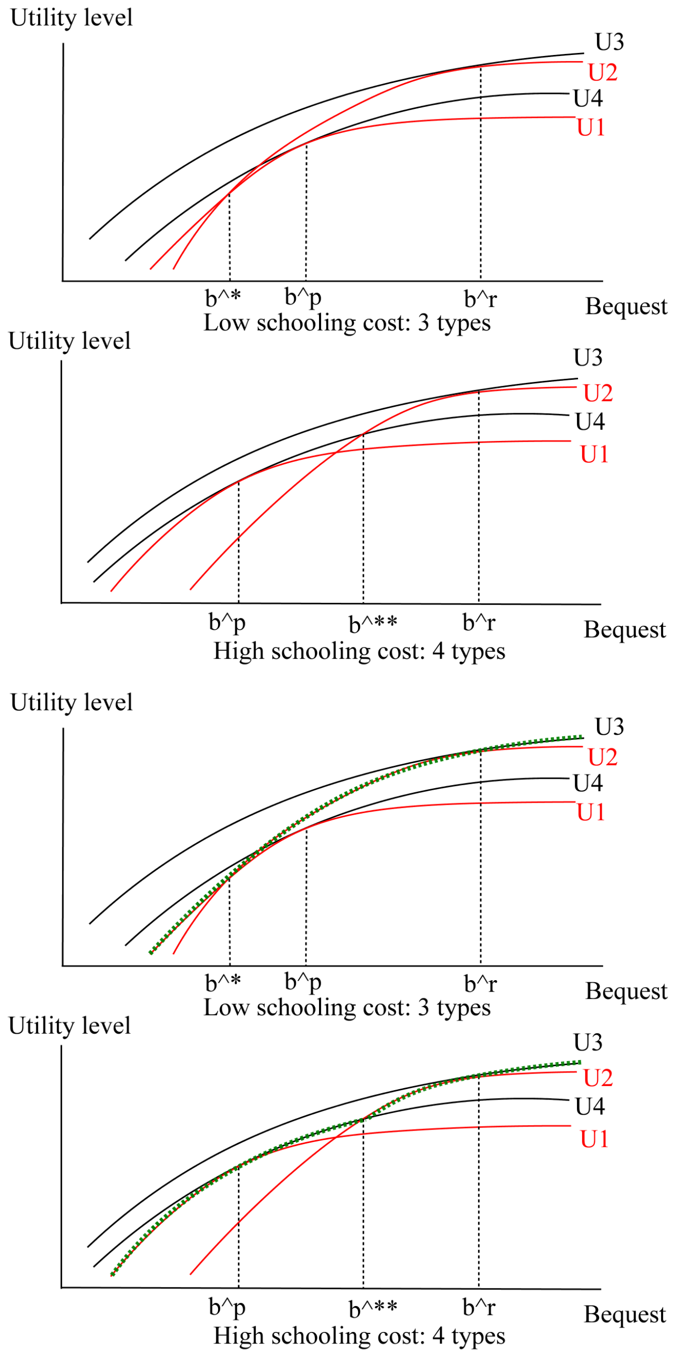


Figure 1: Occupational choice as a function of schooling cost

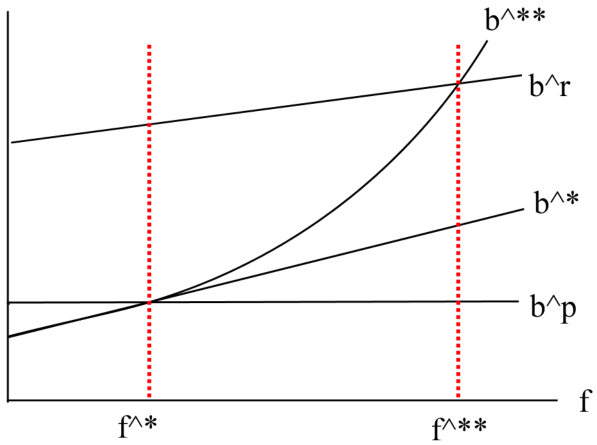


Figure 2: Phases as function of $f$

■

The sole problem is that we have to verify that the following condition is satisfied, $b_t^p < b_t^{**} \leq b_t^r$ (this is necessary for consistency). Numerical analysis shows that the relevant root is $b_1^{**}$ (increasing in $f$, the other being decreasing) and $b_1^{**}$ grows more rapidly than $b^r$ when $f$ increases–for given wages–as can be seen from the Figure (2). Only when $f = f^{**}$ we get $b_t^r = b_t^{**}$. The important point is that when $f = f^{**}$, SPC holds with equality so that no one has a benefit in investment in education. The intuition is that, if the education cost is too high no one would like to invest in it.

The case 2 is interesting: The agents who have a bequest $b^i \in (b^{**}, b^r)$ (type 2 agents) will invest in education while the ones with $\in (b^p, b^{**})$ do not (type 4 agents). The interesting point is that the agents who have a lower bequest $b^i \in (b^p, b^{**})$ have positive savings, while the ones with a higher bequest $b^i \in (b^{**}, b^r)$ do not.

## 4 Wealth dynamics

The capital market equilibrium is such that, at each date, the interest rate must be equal to that of the world:

$$r_t = r \qquad \text{(CME)}$$

The government budget is balanced at each period

$$Z_t + \chi_t H_{t+1} f_t = \tau_t [r S_{t-1} + w_t^H H_t + w_t^L L_t] \tag{GBC}$$

where $S_t = \int_0^1 s_t^i di$. Let us use $n^i$ to represent different types (of agents) according to their saving and skill ($i = L, H, LS, HS$). We can rewrite (LME) as

$$1 = n_t^H + n_t^{HS} + n_t^L + n_t^{LS} \tag{LME$'$}$$

(LME$'$) is the labor market equilibrium. $n_t^L$ is the number of type 1 agents. $n_t^{LS}$ is the number of type 2 agents. In the same way, $n_t^H$ and $n_t^{HS}$ describe respectively type 3 and type 4 agents. The important point is, if $f \leq f^*$ then $n^{LS} = 0$. Since occupational choice is endogenous, the number of each type of workers will also be endogenous.

The objective of this section is to show how one can get $\mathcal{X}_{t+1}$ from $\mathcal{X}_t$ with $\mathcal{X}$ being the vector whose elements are $\mathcal{N}_t, \mathcal{B}_t, \mathcal{T}_t, \pi_t$. $\pi$ is the vector of fiscal instruments, $\pi = \{\chi, \tau, \eta\}$ while $\mathcal{P}$ is the price vector which is given. $\mathcal{B}$ is the bequest vector, $\mathcal{B} = \{b^i\}$. $\mathcal{T}_t$ is the threshold vector, $\mathcal{T} = \{b^p, b^r, b^*, b^{**}, f^*, f^{**}\}$. $\mathcal{N}$ is the labor market vector with $\mathcal{N} = \{n^L, n^{LS}, n^H, n^{HS}\}$. $H$ is scalar.

The price consists of the given factor prices $\mathcal{P} = \{R, w^H, w^L\}$:

$$R = \beta \Gamma k_t^{\beta-1}, \quad w_t^H = (1-\beta)\Gamma k_t^{\beta}, \quad w_t^L = \theta w_t^H$$

As $\mathcal{P}$, the fiscal instruments also are known to the agents, $\pi = \{\chi, \tau, \eta\}$ but decided by the government.

The key equations of thresholds, are now given by

$$b_t^* = \frac{(1-\chi_t) f w^H}{1 - \frac{\omega_{t+1}^L}{\omega_{t+1}^H}}, \quad b_t^p = \frac{\omega_{t+1}^L}{\hat{R}_{t+1}}, \quad b_t^r = (1-\chi_t) f w_{t+1}^H + \frac{\omega_{t+1}^H}{\hat{R}_{t+1}}$$

$$b_t^{**} = \frac{2\,\omega_{t+1}^H - \omega_{t+1}^L - 2\sqrt{\omega_{t+1}^H\,[\omega_{t+1}^H - (1-\chi_t) f w^H\,\hat{R}_{t+1} - \omega_{t+1}^L]}}{\hat{R}_{t+1}}$$

$$f_t^* = \left(1 - \frac{\omega_{t+1}^L}{\omega_{t+1}^H}\right) \frac{\omega_{t+1}^L}{(1-\chi_t) w_{t+1}^H \hat{R}_{t+1}}, \quad f_t^{**} = \frac{(1-\tau_{t+1})(1-\theta)}{(1-\chi_t)\hat{R}_{t+1}}$$

We may write all these relations like

$$\mathcal{T}_t = F_0(\pi_t, \pi_{t+1})$$

Given fiscal instruments–so, the thresholds–one can analyze the evolution of transfers as a function of parental bequests. With imperfect credit markets an individual $i$ will make a bequest which is a function of her parents' bequest. There are two cases, as shown in the preceding analysis.

At any time $t$, $H_t$ is given by the number of agents who had gone to the school in $t-1$. The distribution function of bequests in the society will determine $H_{t+1}$; this is precisely the number of persons who get a bequest $b^i \geq b^*$ in case 1 and $b^i \geq b^{**}$ in case 2. The distribution function of bequests will also determine $n_t^{HS} + n_t^{LS}$, that I define as being the number of people who have positive savings. Another feature of the model is that individuals who do not make a positive saving will bequeath either $b_{t+1}^H$ or $b_{t+1}^L$ which is independent of the amount of the bequest that they have inherited. But, for those who have positive saving it is not the case; our bequest to our offspring is a positive function of the bequest that we got from our parents.

The labor market equilibrium is given by the following equations

$$
\begin{aligned}
1 &= n_t^H + n_t^{HS} + n_t^L + n_t^{LS} \\
H_{t+1} &= n_t^H + n_t^{HS}, \quad n_t^H = \int_{b^{**}}^{b^r} dG_t, \quad n_t^{HS} = \int_{b^r}^{\bar{b}} dG_t \\
L_{t+1} &= n_t^{LS} + n_t^L, \quad n_t^L = \int_{\bar{b}}^{b^p} dG_t, \quad n_t^{LS} = \int_{b^p}^{b^{**}} dG_t
\end{aligned}
$$

these may be rewritten as

$$
\mathcal{N}_t = F_1(\mathcal{B}_t, \mathcal{T}_t)
$$

The evolution of the bequests depends on $f$: If $f \leq f_t^*$,

$$
b_{t+1}^i = \begin{cases} (1-\alpha)\omega_{t+1}^L = b_{t+1}^L \forall i, & \text{if } b_t^i \leq b_t^* \\ (1-\alpha)\omega_{t+1}^H = b_{t+1}^H \forall i, & \text{if } b_t^r \geq b_t^i > b_t^* \\ \frac{(1-\alpha)}{2}\left[\hat{R}_{t+1}(b_t^i - \hat{f}_t) + \omega_{t+1}^H\right], & \text{if } b_t^i > b_t^r \end{cases} \tag{18}
$$

else if $f_t^* < f < f_t^{**}$

$$
b_{t+1}^i = \begin{cases} (1-\alpha)\omega_{t+1}^L = b_{t+1}^L \forall i, & \text{if } b_t^i \leq b_t^p \\ \frac{(1-\alpha)}{2}\left[\hat{R}_{t+1} b_t^i + \omega_{t+1}^L\right], & \text{if } b_t^{**} \geq b_t^i > b_t^p \\ (1-\alpha)\omega_{t+1}^H = b_{t+1}^H \forall i, & \text{if } b_t^r \geq b_t^i > b_t^{**} \\ \frac{(1-\alpha)}{2}\left[\hat{R}_{t+1}(b_t^i - \hat{f}_t) + \omega_{t+1}^H\right], & \text{if } b_t^i > b_t^r \end{cases} \tag{18'}
$$

The case of $f = f_t^{**}$ is special because the agents will be indifferent to their future occupations:

$$b_{t+1}^i = \begin{cases} (1-\alpha)\omega_{t+1}^L = b_{t+1}^L \forall i, & \text{if } b_t^i \leq b_t^p \\ \frac{(1-\alpha)}{2}\left[\hat{R}_{t+1}(b_t^i - \hat{f}_t) + \omega_{t+1}^H\right], & \text{if } b_t^i > b_t^p \end{cases} \tag{18''}$$

And in the last case of $f > f_t^{**}$ there will be only unskilled agents

$$b_{t+1}^i = \begin{cases} (1-\alpha)\omega_{t+1}^L = b_{t+1}^L \forall i, & \text{if } b_t^i \leq b_t^p \\ \frac{(1-\alpha)}{2}\left[\hat{R}_{t+1} b_t^i + \omega_{t+1}^L\right], & \text{if } b_t^i > b_t^p \end{cases} \tag{18'''}$$

All these relations may be represented by

$$\mathcal{B}_{t+1} = F_2(\mathcal{B}_t, \pi_t, \pi_{t+1}, \mathcal{T}_t)$$

And finally the (GBC) can be written

$$Z_t + \chi_t H_{t+1} f_t = \tau_t[rS_{t-1} + w_t^H H_t + w_t^L L_t]$$

or equivalently

$$0 = F_4(\mathcal{N}_{t-1}, \mathcal{N}_t, \mathcal{B}_{t-1}, \pi_{t-1}, \pi_t) \tag{19}$$

The following proposition gathers all this information.

**Proposition 3** *The dynamics of the whole system are given by the following equations given the initial conditions $\mathcal{B}_0$, $\mathcal{N}_{-1}, S_{-1}$.*

$$\mathcal{T}_t = F_0(\pi_t, \pi_{t+1}) \tag{20}$$
$$\mathcal{N}_t = F_1(\mathcal{B}_t, \mathcal{T}_t) \tag{21}$$
$$\mathcal{B}_{t+1} = F_2(\mathcal{B}_t, \pi_t, \pi_{t+1}, \mathcal{T}_t) \tag{22}$$
$$0 = F_4(\mathcal{N}_{t-1}, \mathcal{N}_t, \mathcal{B}_{t-1}, \pi_{t-1}, \pi_t) \tag{23}$$

**Proof.** Given $\{\pi_i\}_0^\infty$, the equation (20) gives the path of thresholds $\{\mathcal{T}_i\}_0^\infty$. Given $\{\mathcal{T}_i\}_0^\infty$, $\{\pi_i\}_0^\infty$ and $\mathcal{B}_0$, the equation (22) gives the whole distribution of bequests, $\{\mathcal{B}_i\}_0^\infty$. Given $\{\mathcal{B}_i\}_0^\infty$ and $\{\mathcal{T}_i\}_0^\infty$, the equation (21) gives $\{\mathcal{N}_i\}_0^\infty$. And finally $\{\pi_i\}_0^\infty$ is chosen by the government such that at each period (23) is respected. ■

**Remark 1** *The initial period, $t = 0$, is special: the equation* (23) *is written* $0 = F_4(\mathcal{N}_{-1}, \mathcal{N}_0, S_{-1}, \pi_0)$*. But, as we see from* (6)*, $S_{-1}$ is related to $\mathcal{B}_{-1}$. Yet, we have assumed that the initial heterogeneity is in $\mathcal{B}_0$, therefore we put* $S_{-1} = \mathcal{B}_{-1} = 0$.

**Remark 2** *If there is no government, the dynamics of the economy are described by* (21) *and* (22)*, because* $\pi_t = \mathcal{T}_t = 0, \forall t$*. So, the relevant initial condition vector is* $\mathcal{B}_0$*. But, as long as there is a government, we need also* (20) *and* (23) *to describe the dynamics of the economy. As a result, the initial condition vectors are now* $\mathcal{N}_{-1}$ *and* $\mathcal{B}_0$*.*

# 5 Numerical analysis

## 5.1 Calibration

Let the interest rate, $r$ be 1 (so, $R = 2$) and $\alpha = .5$. Given the length of the one generation (about 25 years) this imply an annual interest year about 2.81 %. $\beta = .4$ is standard, while $\Gamma = 2.5$ is arbitrary[11]. Hornstein *et al.* (2005) document that $\theta = w^L/w^H$ was variable over time; in fact it was .69 in 1965 while it has reached .588 in 1995 in the United States. They also report that $h = H/(H + L)$, for males, was .15 in 1970 and increased to .3 in 2000; while for females these statistics are respectively .11 and .3. Hendel *et al.* (2005) give similar ratios: in 1965 $h$ was .054 while in 1999 it is .236. So, I will assume $\theta = .6$ and $H_0 = .2$, and $L_0 = .8$ which are close to the average of these values.

There is no evidence about $f$. However, there are two papers that give an idea: the booklet edited by Peretti (2003) for French Youth, Education and Research Ministry, evaluates the total (public plus private) cost/spending of education as 6.9 % of GDP in 2002[12]. Given the labor share parameter, $\beta = .4$ and $\theta = .6$, the total wage bill is $Lw^L + Hw^H = .68w^H = .4Y$ and then $w^H = .4Y/.68$. So, f would be $.069 \times .68/.4 = .40588$. But, as only 10 % of the total spending is made by households, $f$ should be approximatively .04. On the other hand, Jacobs (2002) estimates the yearly cost of university education to be 3900 EUR in Netherlands in 2000 and 2001. If we consider $w^L$ to be the minimum wage, which is approximatively 1000 EUR in European countries, we get $w^H = w^L/\theta \cong 1667$ EUR. This implies $f \cong .195$. The average of these two number is $\cong .117$. I will use different values of $f$ in these two boundary values, i.e. $f \in (.1, .2)$.

In order to simulate the model I also need the initial values of $n^L$, $n^{LS}$, $n^H$, $n^{HS}$ and the initial distribution of bequests. As I do not seek a real calibration, let these values be fixed somehow arbitrarily. I have chosen: $n_0^L = .8$, $n_0^{LS} = 0$, $n_0^H = .15$, $n^{HS} = .05$ and the Poisson distribution with

[11] In fact, one can use $\Gamma$ in order to get the desired value of $k$.

[12] The repartition is the following: 60.7 % government, 22.3 % local authorities, 10 % households, 6.4 % firms and 0.6 % others.

the mean 1.2 for the bequest distribution. There is no special reason for that distribution. I have chosen it because it is right-skewed and discrete. Any right-skewed distribution should give similar results.

An important step of the simulation process is the initial period ($t = 0$). If we follow the standard formulation of the OLG models, like initial bequests, there are also savings which are given. To be consistent with the optimization framework, one could assume that both $b_0^i$ and $s_{-1}^i$ are related in the following way: $b_0^i = (1-\alpha)(\hat{R}_0 s_{-1}^i + \hat{W}_0)$. As we know $b_0^i$, we can get back the right $s_{-1}^i$. But there are two problems: firstly, in the initial distribution there are agents who receive 0 or approximatively zero bequest. The above relation would imply a negative saving, $s_{-1}^i = -\hat{W}_0/\hat{R}_0 < 0$ for these agents. The second important problem is that in order to discuss the efficiency of fiscal and educational redistribution, I fix a constant tax rate and compare the output under fiscal and educational redistribution equilibrium. In such a set-up, increasing the constant tax rate would mean changing the initial aggregate savings. Numerical investigation shows that this change in the initial savings is crucial and affects the whole dynamics of the model. Thus, it becomes impossible to distinguish the effects due to the variation of the initial savings (initiated by a change in the constant tax rate) from the effects of fiscal and educational redistribution. This is why I assume no initial saving, i.e. $s_{-1}^i = 0, \forall i$. See also the Remark (1).

### 5.2 Results

Essentially 4 cases are analyzed: (i) the pure equilibrium case without any government intervention but with low schooling cost; (ii) the pure equilibrium case without any government intervention with high schooling cost; (iii) the equilibrium with only fiscal redistribution; (iv) and finally the equilibrium with only educational redistribution.

The first case is relatively simple. Since the model is one of small open economy, the factor prices are given. With no-intervention as market prices do not change, we can calculate the whole path of all variables for a given initial distribution. The third and fourth ones are a bit more complicated, because, now, the prices/fiscal rates of both the period $t$ and $t+1$ should be taken into account.

I will use GNU Scientific Library[13] (GSL) for numerical work, Maxima[14] for symbolic calculations and CAM Graphics Classes[15] for plotting.

[13]http://www.gnu.org/software/gsl/

[14]http://maxima.sourceforge.net/

[15]http://www.math.ucla.edu/~anderson/CAMclass/CAMClass.html

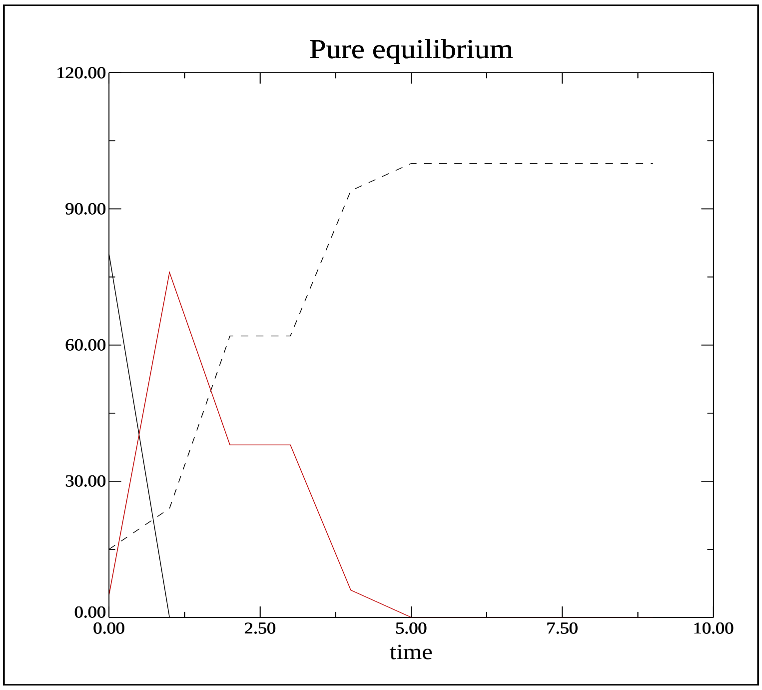


Figure 3: Pure equilibrium with low $f$ ($f = .1$). The black, dashed and red lines show $n^L, n^H$, and $n^{HS}$ respectively. $n^{LS} = 0$ over all the periods.

The Figures (3) and (4) show the importance the schooling cost. They show the evolution of the $n^L, n^{LS}, n^H$, and $n^{HS}$ with respect to time in the case of pure equilibrium, i.e. no intervention case. As I have discussed in previous sections, with a low schooling cost there will be more skilled agents in the economy. When $f = .1$ in the long run all agents are skilled while when $f = .15$ only .76 % are skilled. This result is compatible with Galor and Zeira (1993) who show that when investment in human capital is indivisible and the credit markets are imperfect, initial conditions affect not only the short-run but also the long-run variables[16]. Particularly, I confirm their conjecture according to which, depending on the initial distribution of income, multiple equilibria are possible and that agents may be divided into subgroups. Now on let $f = .15$.

The Figures (5) and (6) show the evolution of the $n^L, n^{LS}, n^H$, and $n^{HS}$

[16]For a similar result of persistent inequality in a slightly different set-up see Ljungqvist (1993).

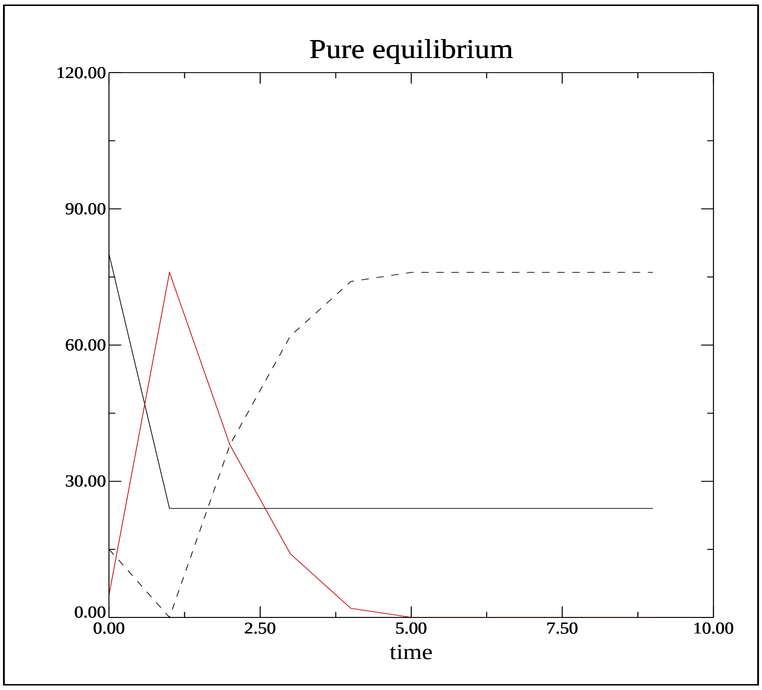


Figure 4: Pure equilibrium with high $f$ ($f = .15$). The black, dashed and red lines show $n^L, n^H$, and $n^{HS}$ respectively. $n^{LS} = 0$ over all the periods.

with respect to time when the tax rate is fixed at $\tau = .0016$ and where all the taxation revenue is spent respectively on educational subsidy and fiscal redistribution. While the educational redistribution is effective at this tax rate (everyone becomes skilled in the long-run equilibrium), the fiscal redistribution is not (only .76 % are skilled). This level of taxation/fiscal redistribution does not suffice to bring economy to a competitive equilibrium with more skilled agents. Intuitively, the borrowing constraints are still binding. In comparison to the pure equilibrium case, the Figure (4), the time path of $n^L$ and $n^{LS}$ are identical to the pure equilibrium one. However, there are fewer unconstrained agents in the transition.

One can wonder if a more higher level of financial redistribution would permit to attain an equilibrium with more skilled agents: the answer is yes. The Figure (7) shows this. The intuition is that the borrowing constraints are no more binding for the constrained agents. A comparison of the Figures (8) and (9) shows this clearly: in the latter one the redistribution rate ($\eta$) is

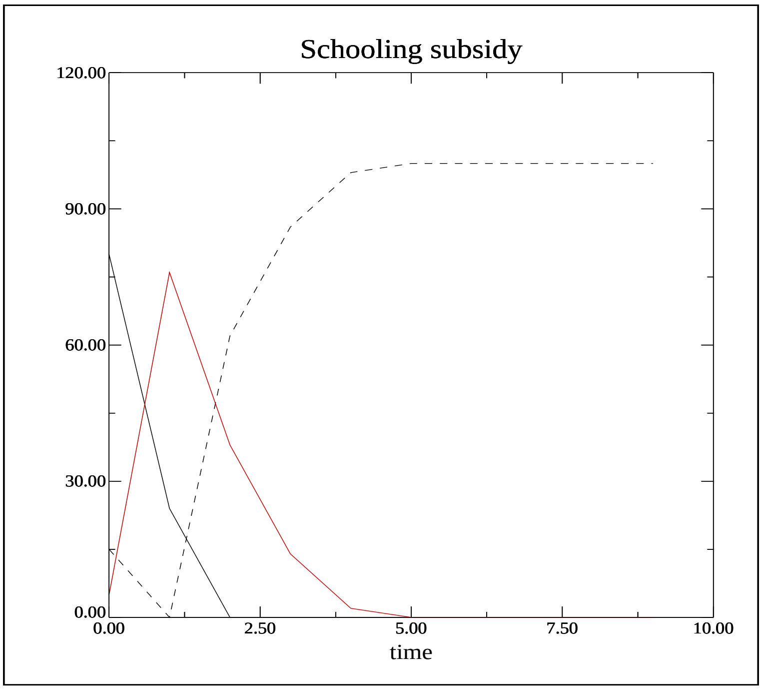


Figure 5: Equilibrium with only educational redistribution for $\tau = .0016$. The black, dashed and red lines show $n^L, n^H$, and $n^{HS}$ respectively. $n^{LS} = 0$ over all the periods.

higher.

To resume, from all these figures, we can say that the educational redistribution is more efficient than the financial redistribution. The intuitive reason is that direct redistribution, on one hand, lessens the borrowing constraints. But on the other hand, it diminishes the incentives for investment in schooling: the ratio of $\omega^L/\omega^H$ increases with $\eta$. This can be also seen from $b^*$; the threshold for the schooling decision. The higher $\eta$ the higher $b^*$.

# 6 Conclusion

I have studied the effect of redistributive taxation in a simple model of investment in schooling where credit markets are imperfect. More precisely future (labor) income can not be used as a collateral for present credit demand. In such a set-up, I showed that family income determines whether

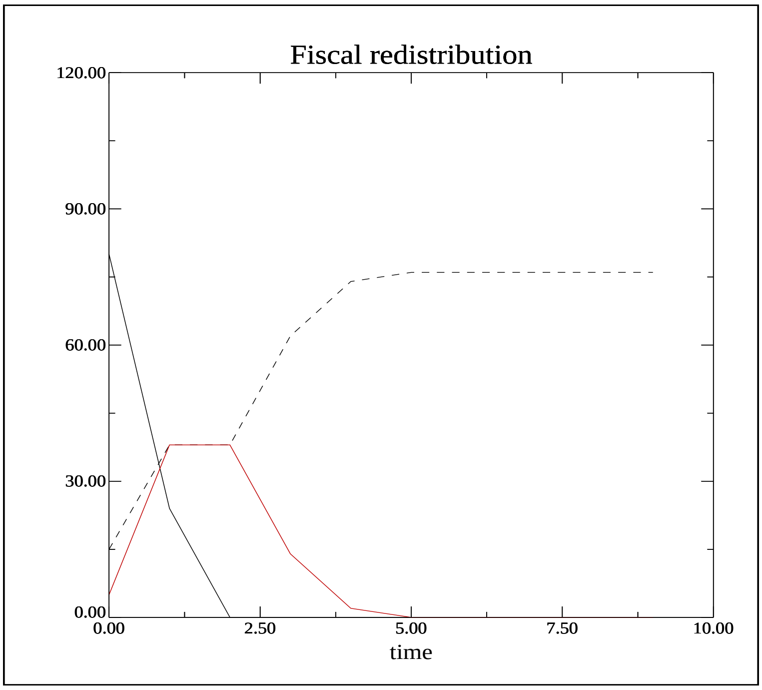


Figure 6: Equilibrium with only fiscal redistribution for $\tau = .0016$. The black, dashed and red lines show $n^L, n^H$, and $n^{HS}$ respectively. $n^{LS} = 0$ over all the periods.

children will invest or not in schooling and so whether they will be qualified or not. After studying the dynamics of the model I have a given a numerical example which characterizes the role of fiscal and educational redistribution financed by distortive taxation.

In comparison with no intervention case, distortive taxation that is used to finance the subsidy to education increases the ratio of skilled labor. This is also true for the financial redistribution but, as our example illustrated, in order to create the same effect the tax rate should be higher (if the only fiscal instrument is financial redistribution). The intuition is that the educational subsidies are more efficient than the direct redistribution in dealing with borrowing constraints that prevent the poor agents to invest in education.

To have a simple and manageable model I have used a few simplifying assumptions. More realistic assumptions would strengthen the main message. One would like to extend the model by introducing, for example, the stochas-

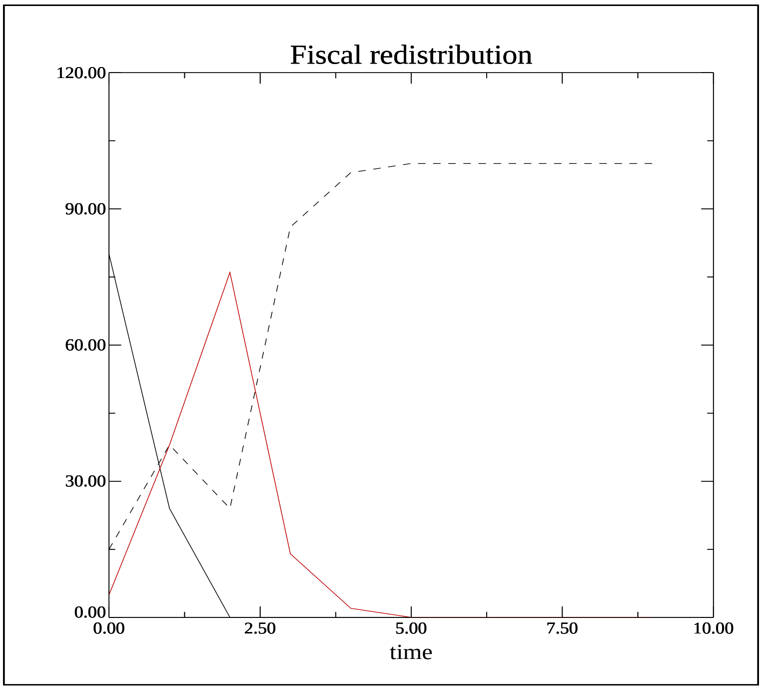


Figure 7: Equilibrium with only fiscal redistribution for $\tau = .005$. The black, dashed and red lines show $n^L, n^H$, and $n^{HS}$ respectively. $n^{LS} = 0$ over all the periods.

tic ability; one another may think about closed economy. Both are important steps that will enrich the dynamics of the model by allowing more mobility. A final extension may be considering the endogenous growth framework.

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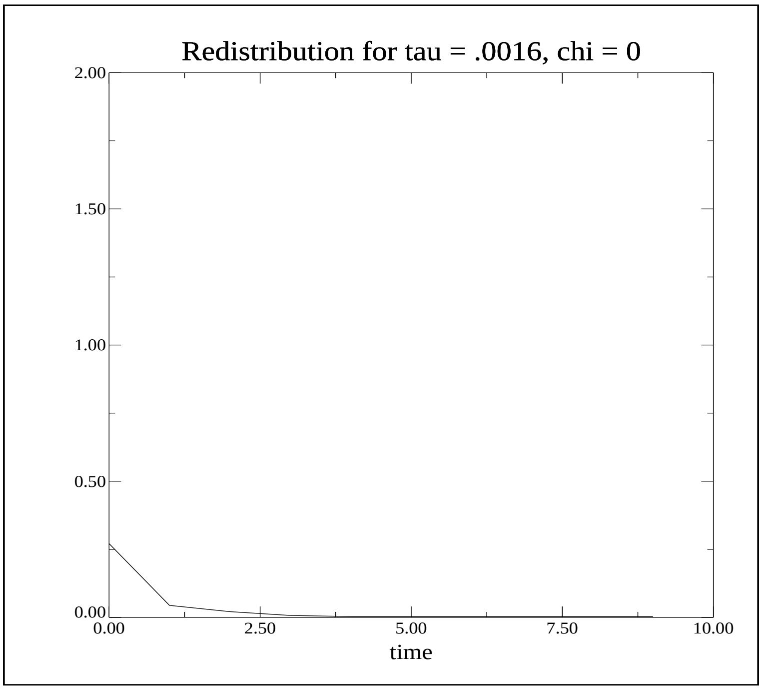


Figure 8: Implied redistribution level ($\eta$) for $\tau = .0016$.

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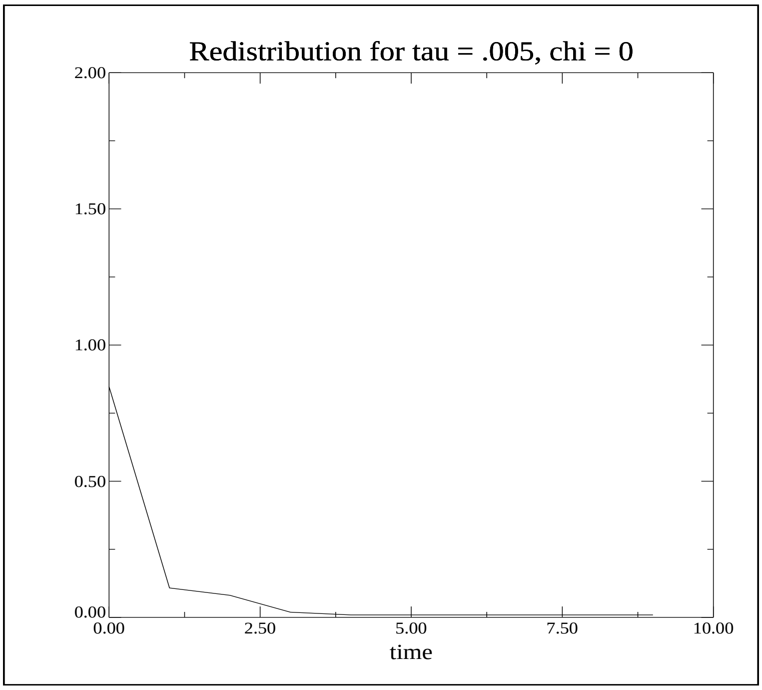


Figure 9: Implied redistribution level ($\eta$) for $\tau = .005$.

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